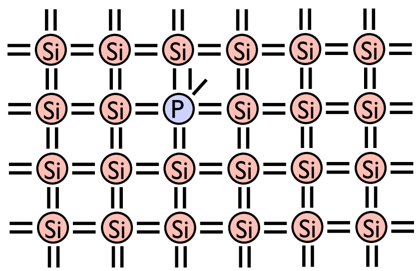
# OPERATIONAL AMPLIFIERS AND ANALOG COMPUTERS

## SECTION 1: Ode to the seemingly useless device . . . or, even apparently dumb ideas can prove to be useful.

Take an insulator, a host material like silicon, and replace every 10,000th silicon atom with one phosphorous atom.

Because phosphorous has one more valence electron than does silicon, you will be left with a "doped" structure that has "extra" electrons floating around within it.

If you put the structure in an electric field (it actually even happens without and at room temperature), you will find those free, negatively charged electrons migrating through the structure. This kind of material is called an n-type semiconductor.

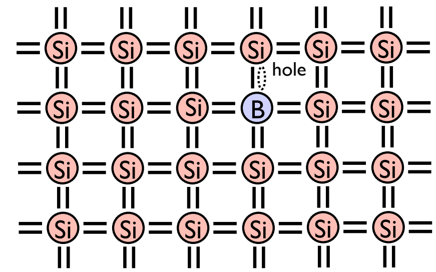





Take an insulator host material like silicon and replace every 10,000th silicon atom with one boron atom.

Because boron has one fewer valence electrons than does silicon, you will be left with a "doped" structure that has " electron holes"—places where electrons should be but aren't—floating around within it. These holes will appear to be electrically positive.

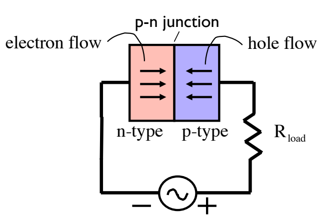
If you put the structure in an electric field, you will find those free, "positive" holes migrating through the structure. This kind of material is called a p-type semiconductor.

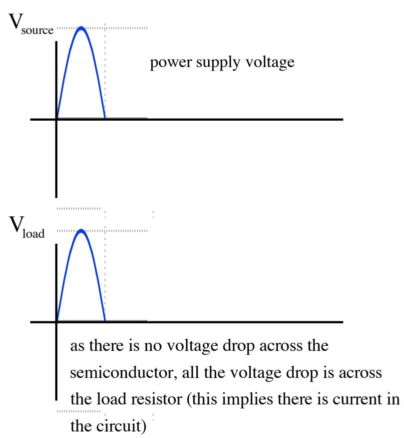





So what happens if we "glue" a p-type semi-conductor onto an n-type semi-conductor, then place the structure across an AC power supply?

When the polarity is as shown in the sketch, the power supply's electric field will drive the negative electrons in the n-type semiconductor to the right (look at the graphic!) and the "positive" holes in the p-type semiconductor to the left.

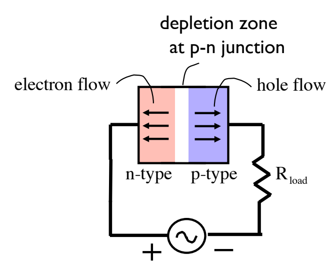


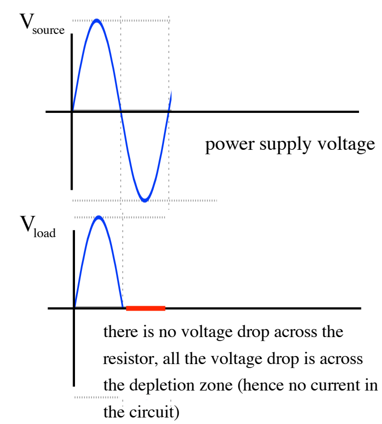




The holes and electrons will combine at the p-n junction, no voltage drop will occur across that junction and all the voltage drop will happen across the load resistor. In other words, there will be current in the counterclockwise direction in the circuit.

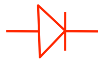


When the power supply polarity changes, the electrons in the n-type semiconductor will move to the left while the holes in the p-type semiconductor will move to the right. This will produce a depletion zone at the p-n junction. Acting like a break in the circuit, all the voltage will drop across the junction so that no voltage drop occurs across the resistor. That means *NO current in the circuit.*

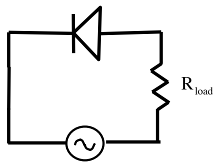


This device is called a DIODE. It is designed to turn AC into DC.

Its symbol is: 

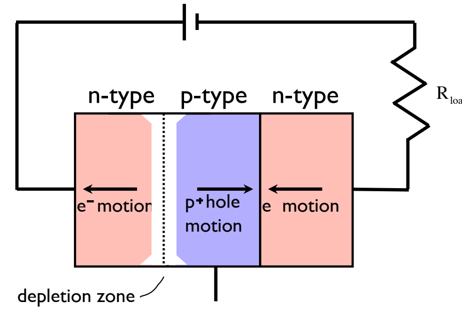
To be complete, the arrow points in the direction of acceptable current flow. For the circuit we just looked at where the expected current is counterclockwise, a circuit schematic would look like:




So talk about useless, what do you suppose would happen if we took two diodes, glued them back-to-back and placed them across a DC power supply?



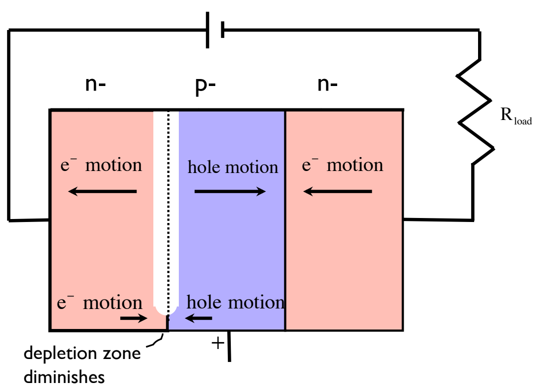
Following the reasoning we've already developed, it doesn't take a genius to see that we would find a depletion zone across one of the p-n junctions (see sketch) and no current would flow through the load resistor.




BUT WHAT IF WE WERE CLEVER?

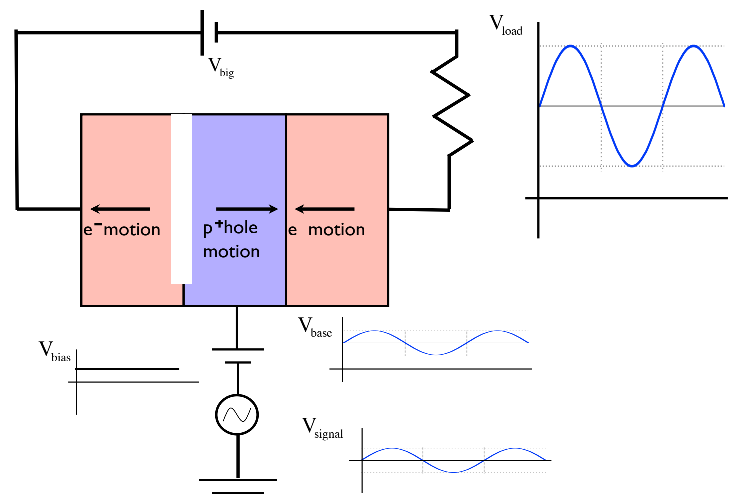


If we attached a terminal to the middle section and made it electrically positive, electrons at the bottom of the left n-type semiconductor would be attracted rightward (look at the sketch) and holes at the bottom of the p-type semiconductor would be repulsed leftward and the depletion zone at the bottom of the p-n junction would diminish effectively allowing "current" to flow through the circuit and, as a consequence, the load resistor. What's more, the degree of positiveness at the base terminal would govern the SIZE of the current through the load resistor.



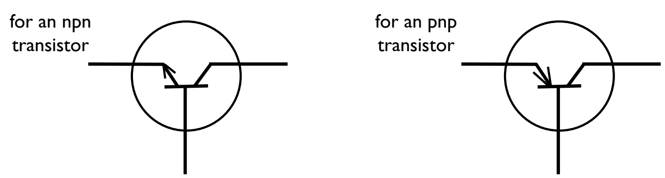



In other words, as long as you keep the base positive, any variation of base voltage will produce the exact same variation in the load resistor circuit, except BIGGER. In my country, we call this an amplifier.



This amplifying device, which started out looking useless, is called a *transistor*. It is the basis of nearly all of today's amplified sound systems!

And just so you know, should you ever run across one, the circuit symbol for a transistor is:






The material you are about to run into was generated primarily from the ebook Lessons In Electric Circuits – Volume III (Chapter 8).

This text can be found at

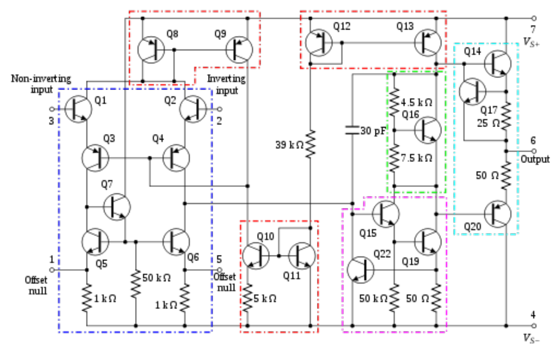
http://openbookproject.net/electricCircuits/Semi/index.html



## SECTION 2: Another seemingly dumb ideas, the OP AMP.

So consider the circuit shown to the right (note all the transistors in it).

Whereas transistors have single terminal inputs (you put in a signal, it puts out an amplified version of the signal), this rather ungodly looking thing is the inner working of an operational amplifier. It has *dual* terminal inputs.

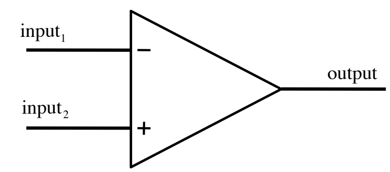



Op amps are designed to do a very simple thing. It is that which we are about to talk.

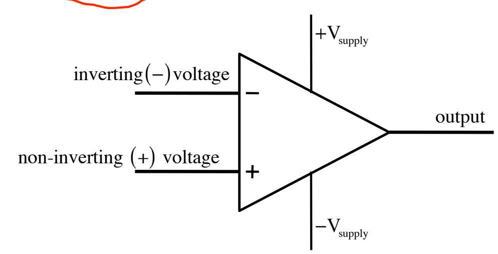


## The Differential Amplifier:

A simplified presentation of a *differential amplifier* is shown to the right. In a nutshell, it takes the difference between the two inputs and outputs that difference amplified.




Extremely high gain differential amplifiers (gain in the 200,000 to 250,000 range) are called *OPERATIONAL AMPLIFIERS*, or OP AMPS. Their symbol is shown to the right. Their inputs are given special names, shown on the sketch, and I've included on the sketch the external power supply leads needed to run the device.



## So how are these used? We will start at a crawl:

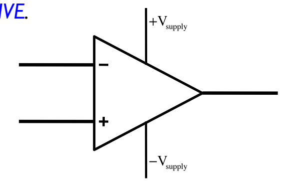
All *differential amplifiers* work on the same principle.

a.) As was said, the DIFFERENCE between the input voltages is amplified, and that amplified result becomes the output. Here are the rules:

i.) If the voltage of the inverted input (the – terminal) is greater than the non-inverted input (the + terminal), the output is *NEGATIVE*. (Shouldn't be a surprise!)

ii.) If the voltage of the inverted input (the – terminal) is less than the non-inverted input (the + terminal), the output is *POSITIVE*.

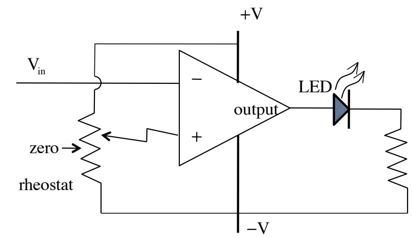
b.) The output is limited by the supply voltages (note that I will not include those terminals from here on).





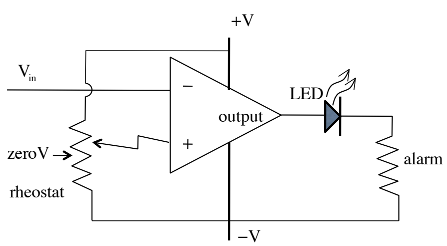

c.) Let's assume the power supply voltages (the +V and –V), hence output voltages, range between +15 and -15 volts. If the gain is 200,000, it only take a 75 microvolt potential difference (that's .000075 volts) between the + and – terminals to generate the maximum 15 volt output. In other words, if the two input voltages are not really, really, really close, the amplifier saturates and you ALWAYS get the maximum output.

This probably doesn't seem very useful, having an amplifier that is always generating its maximum output that's either positive or negative, but consider the circuit shown to the right.



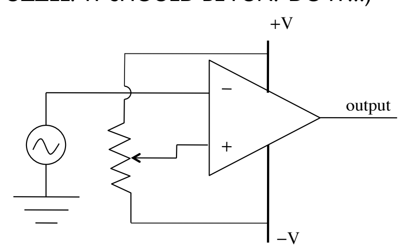


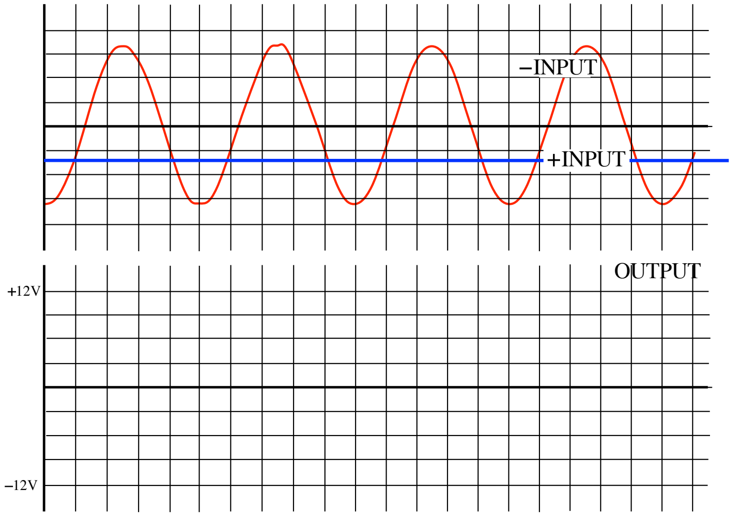

There are instances when you might not want the voltage in a circuit to drop below a certain value (maybe it's a cooling system that is critical to the operation of machinery because at lower voltage, the cooling system doesn't adequately do its job). If the voltage drops to a point that is too low, you want a warning light and alarm to go off. In this circuit, current will not flow through the warning light (LED) and alarm unless current *can* flow through the LED, and that will only happen if the op-amp's output voltage is positive. Positive voltage output for the op-amp only happen if the "-" terminal is lower voltage than the "+" terminal, which is a value you can set using the variable resistor (the rheostat). So you set the rheostat to the positive voltage below which you don't want the system to go, and the op-amp will continuously output a negative maximum voltage until the input voltage gets too low whereupon the output goes positive and the alarm goes off. Problem solved!



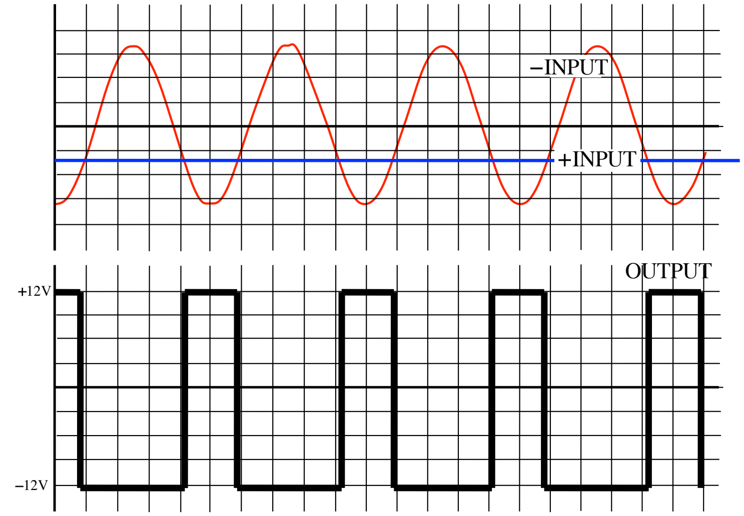



Another example: Consider the circuit shown below. Actually take out a piece of paper, draw an axis, draw in the "+" terminal voltage, draw in the "-" terminal voltage (assuming this hasn't been provided), then draw in what you think the output voltage will look like. REMEMBER, this is a differential amplifier—it is taking the DIFFERENCE between the two input terminals, amplifying them hugely (most probably to saturation, which will be + or – 12 volts for this case, depending upon which input terminal voltage was larger), and that's the output. THOSE ARE THE RULES. FOLLOW THEM AND SEE WHERE THEY TAKE YOU!!! (IN OTHER WORDS, THIS IS A PUZZLE. IT SHOULD BE FUN. DO IT!!!)





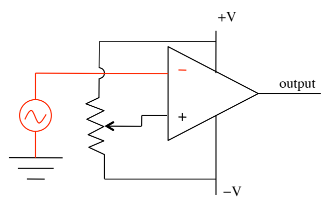


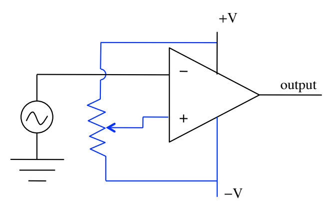



Looking at it in pieces.

a.) To begin with, the inverter (-) terminal has a sine wave coming into it.

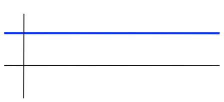



b.) There is a voltage across the rheostat (the resistor) generated by the amplifier's power supply terminals (the +V and –V terminals). The center tap selects what part of that voltage will go into the non-inverter (+) terminal.

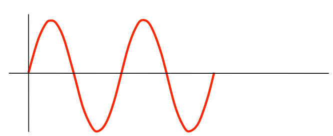



c.) So:

+ terminal's input looks like:

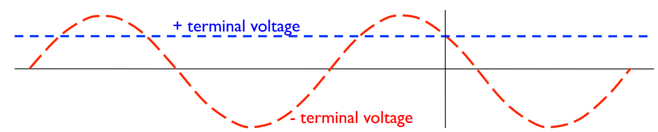


- terminal's input looks like:

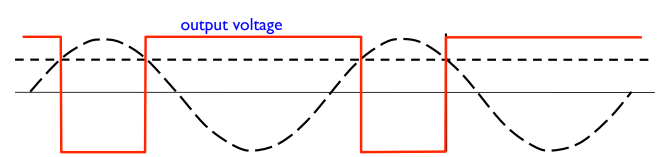




d.) Superimposing the two voltages on top of one another, we get:

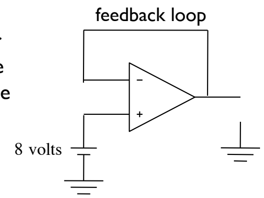



e.) The op amp's output is generated by taking the difference between the two terminals and multiplying it by 200,000. That will put the output at its maximum almost always. Remember that when the "+ terminal's voltage" is <u>GREATER </u>than the "– terminal's voltage," the output will be maximum and <u>positive</u>, and when the "+ terminal's voltage" is less the output will be maximum and <u>negative</u>. The result is a square wave whose duty cycle is governed by the voltage set by the rheostat.




This is NOT useless!

# FEEDBACK




Assume that an 8 volt battery is connected to the non-inverter (+) input terminal of an op amp whose gain is 200,000. Assume also that a line is used to connect the output on the right to the inverter(-) terminal as shown in the sketch (this is called a *feedback loop*).

The amplifier is turned on. Once the amplifier settles down, we know that the output will be the same as the the inverter's input, the value of which we can call "x." We also know that 200,000 times the difference between "x"and 8 must equal the output, or "x." In other words,

$$200{,}000(x-8)=x$$

$$\Rightarrow \quad 200{,}000x - x = 200{,}000(8)$$

$$\Rightarrow \quad x = \frac{200{,}000(8)}{199{,}999}$$

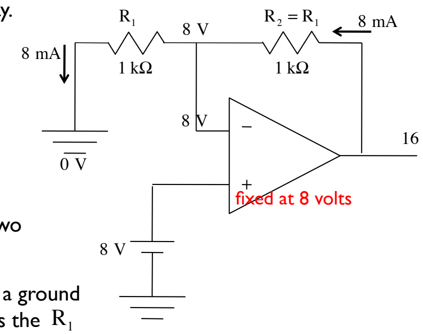
$$\Rightarrow \quad x = 7.99996 \approx 8$$

In other words, with the feedback we are assured that – (inverter) voltage will essentially be the same as + (non-inverter) voltage. NOT USEFUL YET, BUT GETTING THERE!



Let's now modify our *feedback* circuit as show in the sketch.

OK, this looks awful. Follow along anyway.




a.) Using a battery, we have fixed the + (non-inverted) input at 8 volts.

b.) With the feedback look, we know that the – (inverted) input will also be approximately 8 volts.

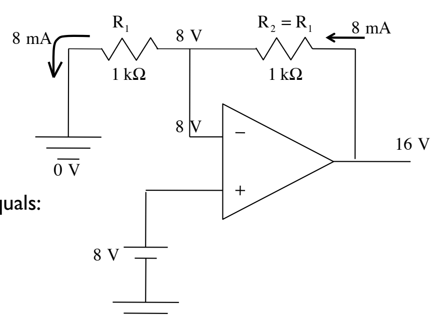
c.) That means the point between the two resistors will have a voltage of 8 volts.

d.) With an 8 volt point on its right and a ground connection on its left, the voltage across the $R_1$ resistor will be 8 volts and the current through that resistor (from Ohm's Law) will be: $V_{R1} = i_1R_1$

$$\Rightarrow \quad i_1 = V_{R1}/R_1$$

$$= (8\text{volts})/(10^3\Omega) = 8\text{ mA}$$






e.) Because no current will flow into the (–) terminal (its impedance—it's resistance to charge flow—is enormous, so practically no current will flow into it), that same 8 mA will also flow through $R_2$.

f.) The voltage difference across $R_2$ will equals:

$$V_{R2} = i_2R_2$$

$$= (8x10^{-3}A)(10^3\Omega)$$

$$= 8\ V$$

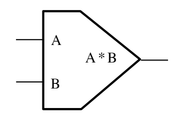
g.) If the voltage on the left side of $R_2$ is 8 volts, and the voltage difference across $R_2$ is 8 volts, the voltage on the output side must be16 volts.

h.) (Notice that by changing the resistor values, the output value can be altered.)

i.) Bottom line: This op amp set-up allows us to generate at the output that is a MULTIPLE of the input voltage.

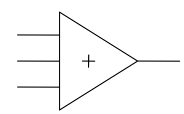


Op Amps that are set up to do this are called MULTIPLIERS. Their circuit symbol is shown to the right. Note that one input will be what is being multiplied, and the other is by how much (this will make more sense shortly).

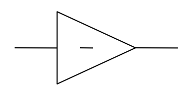



Beyond the MULTIPLIER, there are five other circuit configurations that op amps can be turned into. Three of them are:

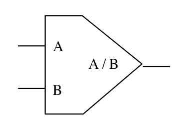
a.) The SUMMER (their output is the *sum* of several inputs);




b.) The INVERTER (their output is *minus* their input);




c.) The DIVIVER (their output is some fraction of their input).




The last two, the INTEGRATOR and DIFFERENTIATOR , are worthy of a more complete look.

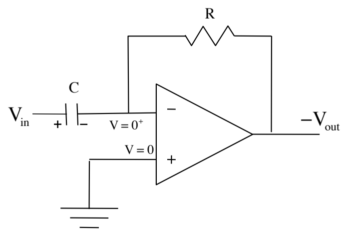
# DIFFERENTIATOR

A differentiator is an op amp circuit whose output voltage is proportional to the time *derivative* of the input voltage (that is, $V_{out} \propto {}^{dV_{in}}\!/_{dt}$ ). How so?

a.) We set the + (non-inverter) terminal to zero voltage by connecting it to a ground connection. With the feedback, that means the – (inverter) terminal will also be approximately zero volts.

b.) This means the voltage across the capacitor will be $V_{in}$ .

c.) Using the definition of capacitance, we can write:




$$C = \frac{q_{oncap}}{V_{cap}}$$

$$\Rightarrow \quad q_{oncap} = CV_{cap}$$

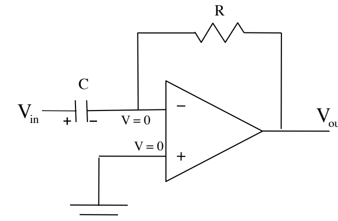
$$\Rightarrow \quad \frac{dq_{oncap}}{dt} = C\frac{dV_{cap}}{dt}$$

$$\Rightarrow \quad i_{thrucap} = C\frac{d(V_{in})}{dt}$$



d.) With the voltage on the left side of the resistor equal to zero, the voltage across R is $-V_{out}$.

e.) Using the Ohm's Law on the resistor:




$$i_R = \frac{-V_R}{R}$$

f.) But because the inverter input has high impedance (high resistance to current flow), the current through the resistor and capacitor will be the same and we can write:

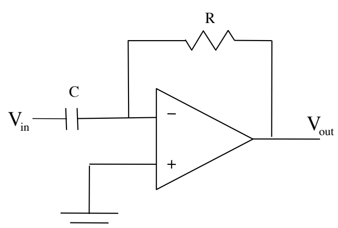
$$i_R = \frac{-V_{out}}{R} = i_{thrucap} = C\frac{dV_{in}}{dt}$$

$$\Rightarrow \quad \frac{V_{out}}{R} = -C\frac{dV_{in}}{dt}$$

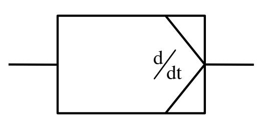
$$\Rightarrow \quad V_{out} = -RC\frac{dV_{in}}{dt}$$



g.) Bottom line: $V_{out} = -RC\frac{dV_{in}}{dt}$ tells us that the output voltage is proportional to the derivative to the input voltage. Additionally, the gain is equal to the -RC of the op amp circuit and a phase shift of $180^o$ exists between the input and output voltages (that last point is what the negative sign is telling us). This element, in theory, should work for any input frequency, though it apparently has instability at in the higher frequency range.




h.) The schematic symbol for an op amp acting as a differentiator is shown to the right.

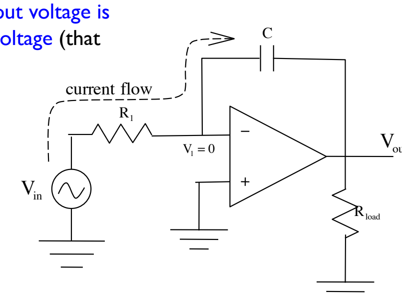





# INTEGRATOR

An integrator is an op amp circuit whose output voltage is proportional to the *time integral* of the input voltage (that is, $V_{out} \propto \int V_{in} dt$ ). How so?

a.) As before, the –(inverted) and +(non-inverted) terminals are set to zero volts.



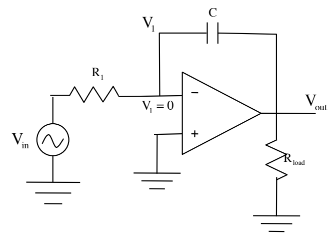

b.) With the voltage on the right side of $R_1$ being zero and the voltage on the left side being $V_{in}$, the voltage difference across the resistor will be $V_{in}$ and the current through the resistor (by Ohm's Law) will be:

$$i_R = \frac{V_{in}}{R_1}$$

c.) Because the input impedance (resistance) of the inverter (-) input is very high, essentially all of the current coming from the input voltage $V_{in}$ will pass through both the resistor $R_1$ and the capacitor C.

d.) We established earlier when dealing with the differentiator circuit that the current through C is

$$i_{thrucap} = C\frac{d(V_C)}{dt}$$




e.) As the current through the resistor and the capacitor is the same, we can write

$$i_{thrucap} = C\frac{d(V_{out})}{dt} = i_R = \frac{V_{in}}{R_1}$$

$$\Rightarrow \frac{V_{in}}{R_1} = C\frac{d(V_{out})}{dt}$$

$$\Rightarrow \frac{V_{in}}{RC}dt = d(V_{out})$$

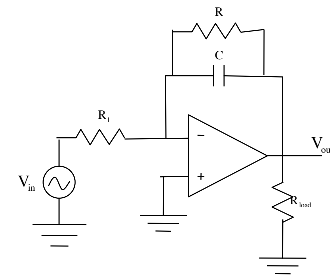
f.) If we integrate both sides, we get:

$$\frac{1}{RC}\int V_{in}dt = \int d(V_{out})$$

$$\Rightarrow V_{out} = \frac{1}{RC}\int V_{in}dt$$

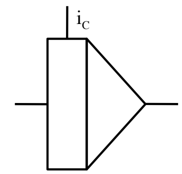


h.) Bottom line: The output voltage of the op amp will be proportional to the integral of the input voltage $V_{in}$ with a gain of $1/R_1C$ .




i.) Note, to be complete: There are problems that arise with low gain situations in a circuit like this. The additional resistor across the capacitor (see sketch) is there to deal with this. Additionally, the gain is not stable over all frequencies. This is dealt with using an advanced design.

j.) The schematic symbol for an op amp acting as an integrator is shown to the right.

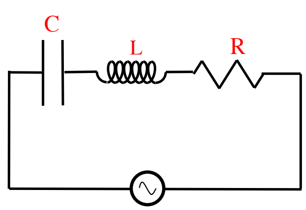





# THE PAYOFF

So now it's time to look back at our RLC circuit—the one we decided to use to simulate our spring driven oscillating cart—but with a twist. We are going to assume there is a variable speed motor in the spring system that motivates the system to oscillate.

The electrical equivalent of a variable motor in our oscillating spring system is an AC voltage source. The circuit is shown to the right.




The differential equation we are going to simulate is:

$$\frac{1}{C}q - (-\dot{q})R + L\ddot{q} = V(t)$$

$$\Rightarrow \ddot{q} + \left(\frac{R}{L}\right)\dot{q} + \left(\frac{1}{LC}\right)q = \frac{V(t)}{L}$$



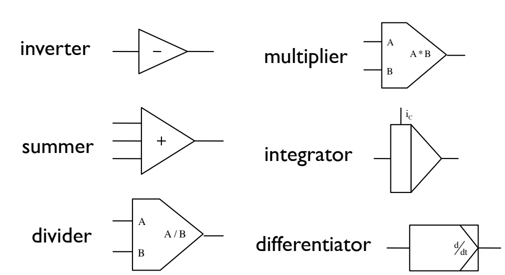
The time derivative of this equation yields:

$$\frac{d\left(\ddot{q} + \left(\frac{R}{L}\right)\dot{q} + \left(\frac{1}{LC}\right)q\right)}{dt} = \frac{1}{L}\frac{dV(t)}{dt}$$

$$\Rightarrow \ddot{I} + \left(\frac{R}{L}\right)\dot{I} + \left(\frac{1}{LC}\right)I = \left(\frac{1}{L}\right)\frac{dV(t)}{dt}$$

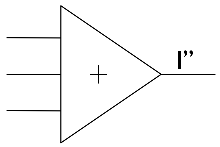
$$\Rightarrow I'' = -\left(\frac{R}{L}\right)I' - \left(\frac{1}{LC}\right)I + \left(\frac{1}{L}\right)V'$$

This requires a summing circuit that includes an "I" (current) term, its derivative and its second derivative. See if you can draw such an operator, correctly labeled, before I show it to you. Your schematic options are shown to the right



$$I'' = -\left(\frac{R}{L}\right)I' - \left(\frac{1}{LC}\right)I + \left(\frac{1}{L}\right)V'$$

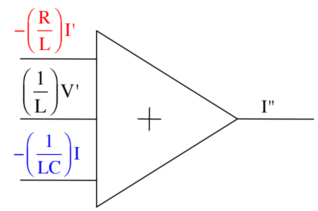
OK—your summing, so start with a summer. I'll get you started:



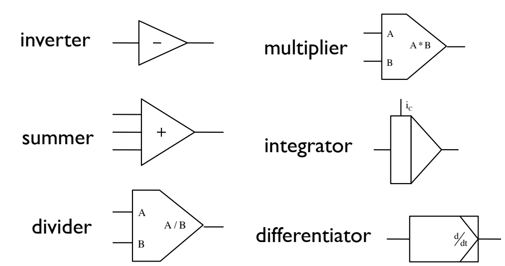



$$I'' = -\left(\frac{R}{L}\right)I' - \left(\frac{1}{LC}\right)I + \left(\frac{1}{L}\right)V'$$

So the summing circuit looks like:

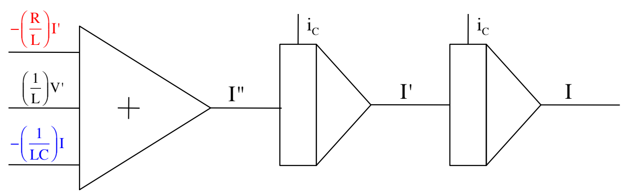



Now add whatever is needed to that element to get I' and I using one or more of the elements to the right?

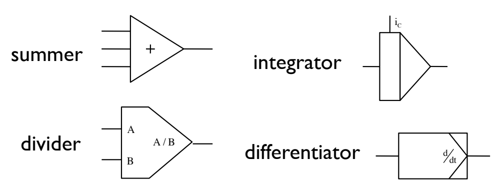




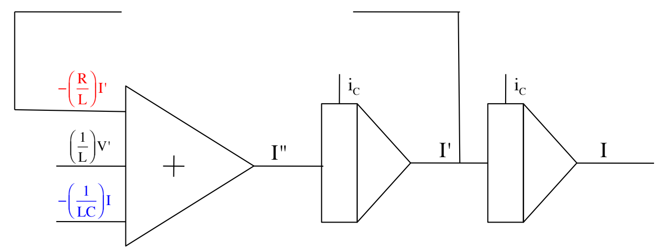

Integrators will do the trick. That is:




Now, again using one or more of the elements to the right, what would you have to do to a lead from the I' section to get (-R/L). That is, how are you going to get R/L times I', and how are you going to get the negative sign? Again, think about the elements you have available:

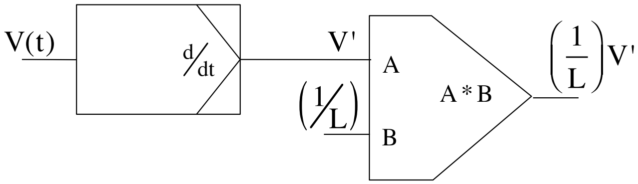





Now, how are you going to get (1/L)V'? You can assume you have a power supply V(t).

The element combination that does the job is:

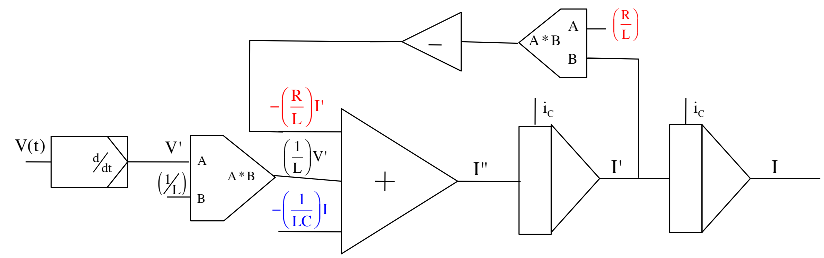



Putting it all together, we get:



So far:

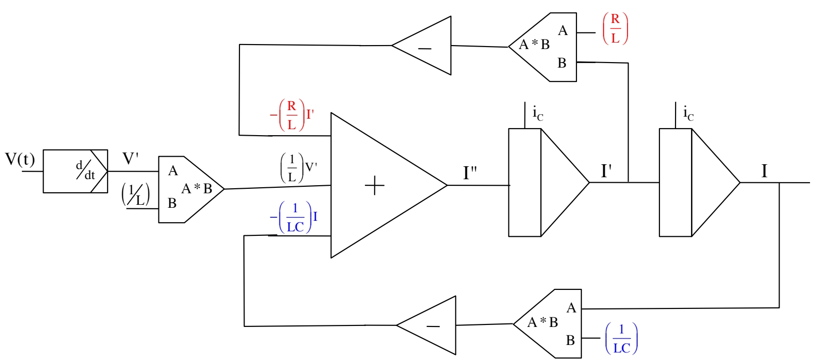
$$I'' = -\left(\frac{R}{L}\right)I' - \left(\frac{1}{LC}\right)I + \left(\frac{1}{L}\right)V'$$




Now finish it on the circuit above:



Building on what we had, that last piece completes the circuit.




This is the op amp circuit needed to analyze our differential equation. All we need to do is input the voltage function, and the current will be proportional to the solution of our differential equation.

COOL, EH?



Assignments 4: Now for the real fun! In the Electromechanics PowerPoint pdf, we concluded that the differential equation for our spring system

$$\ddot{x} + \left(\frac{D}{m}\right)\dot{x} + \left(\frac{2k}{m}\right)x = 0$$

corresponded to the differential equation for an RLC electrical circuit.

$$\ddot{q} + \left(\frac{R}{L}\right)\dot{q} + \left(\frac{1}{LC}\right)q = 0$$

To wire an op amp circuit for this situation, we had to get the variables into a form we could use, so we took the time derivative to get current terms (i's, i dots and i double dots). With the relationship in terms of current, we were able to wire the analog computer.

The differential equations for our spring system could also have been written in terms of velocity. In fact, that relationship is shown below.

$$\dot{v} + \left(\frac{D}{m}\right)v + \left(\frac{2k}{m}\right)\int v\,dt = 0$$

a.) Begin with the equation

$$\dot{v} + \left(\frac{D}{m}\right)v + \left(\frac{2k}{m}\right)\int v\,dt = 0$$

and substitute in the electrical counterparts for each variable.

b.) Once you have the equation, lay out the schematic for an analog computer circuit using summers and integrators and differentiators, etc., that would model your circuit. (Note that this will look something like the circuit shown on Slide 39).

THIS SHOULD BE FUN (kind of like a puzzle)! Revel in the thought of all of those ferns and trees you're about to grow!